

A model with bulk and brane gauge kinetic terms and SUSY breaking without hidden sector

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Abstract

We derive various solutions for the gauge field and the gaugino when there are both 5D bulk kinetic terms and 4D brane kinetic terms. Below the compactification scale $1/y_c$, gauge interaction by the massless mode is universal, independent of the locations of sources, but above $1/y_c$, the interaction distinguishes their locations. We consider the $S^1/(Z_2 \times Z'_2)$ orbifold compactification, in which $N = 2$ SUSY and $SU(2)$ gauge symmetry break down to $N = 1$ and $U(1)$, respectively. While the odd parity gauge fields under Z_2 , A_μ^1 and A_μ^2 , interact with the bulk gauge coupling g^2 , A_μ^3 at low energy interacts with $e^2 g^2/(e^2 + g^2)$ due to the brane kinetic term with the coupling e^2 . Even if g^2 , which is asymptotically free, blew up at the scale $1/y_c < \mu < \Lambda_{\text{cutoff}}$, $e^2 g^2/(e^2 + g^2)$ could remain small at low energy by brane matter fields' contribution. The condensations of the gauginos Ψ^1 and Ψ^2 generate soft mass term of Ψ^3 by gravity mediation.

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1 Introduction

For the last seven years, the brane world idea has been one of the major research fronts in particle physics, because the idea provides possibilities to resolve many theoretically difficult problems in particle physics in geometrical ways. In the earlier stage of the brane revolution, it was discussed that the brane scenario could be applied to resolve various scale problems like the gauge hierarchy problem [1, 2] and the μ problem [3]. In the setup where all kinds of the standard model fields live on a 4D brane while gravity can propagate in the whole higher dimensional space-time (bulk), the fundamental scale may be lowered to the grand unification scale or even to TeV scale by assuming large volume of the extra dimensional space. Such a setup was utilized to resolve the gauge hierarchy problem [1] and the string-GUT scale unification problem [4].

Recently, it was pointed that the orbifold compactification gives an efficient mechanism for breaking supersymmetry (SUSY) and/or gauge symmetry in the context of the higher dimensional supersymmetric grand unified theory (GUT) [5]. In orbifold compactifications, a presumed discrete symmetry of the extra dimensional space is imposed to the field space. Then, only invariant bulk fields under the symmetry survive at low energy, and can couple to brane fields on lower dimensional orbifold fixed points (brane). In $S^1/(Z_2 \times Z'_2)$ orbifold compactification models, the original gauge group \mathcal{G} that the bulk Lagrangian respects breaks down to a sub-group \mathcal{H} commuting with Z_2 , and only one half of the supersymmetries introduced in the bulk action survive at low energy because of the other Z'_2 . Then, the required gauge symmetry on the branes is \mathcal{H} rather than \mathcal{G} [6]. A resolution to the doublet-triplet splitting problem in GUT employing this mechanism was proposed in [5].

In this paper we will study extensively the model in which the kinetic terms of the gauge field(s) and gaugino(s) with even parity are present both in the bulk and on the brane(s) (Z_2 orbifold fixed point(s)), in the $S^1/(Z_2 \times Z'_2)$ orbifold compactification. Only if the brane kinetic term respects $N = 1$ SUSY and the gauge symmetry \mathcal{H} , are

such terms harmless. Moreover, brane kinetic terms, generally, could be generated radiatively on the brane, even if it is absent in the tree level action [7]. In this paper, however, we will assume the brane kinetic term is not small from the beginning. In this setup, we will derive the gauge field and the gaugino propagators, and discuss a SUSY breaking scenario with the propagators.

2 Gauge field solutions

In the presence of the kinetic terms of the gauge field and gaugino on the brane at $y = 0$ as well as in the bulk, the action is

$$S = \int d^4x \int_{-y_c}^{y_c} dy \left[\frac{1}{2y_c} \frac{-1}{4g^2} F_{MN} F^{MN} + \delta(y) \frac{-1}{4e^2} F_{\mu\nu} F^{\mu\nu} - \left(A^M J_M \right) + \frac{1}{2y_c} \frac{i}{g^2} \bar{\Psi} \not{D}_5 \Psi + \delta(y) \frac{i}{e^2} \bar{\Psi} \not{D}_4 \Psi - \left(\bar{\Psi} \eta + \bar{\eta} \Psi \right) \right], \quad (1)$$

in which J_M and η are vector-like and fermionic sources that are coupled to the gauge boson and gaugino, respectively. g^2 and e^2 are the couplings of the bulk and brane gauge kinetic terms. In this paper, M, N indicate 5D space-time, $(x^\mu, x^5 = y)$, where $\mu = 0, 1, 2, 3$. When the source is localized on the brane, $J_M(x, y) = \delta_M^\mu J_\mu(x) \delta(y)$, the equation of motion of the gauge field, derived from the action (1) is [7]

$$\left[k^2 + \partial_y^2 + \delta(y) \frac{2y_c g^2}{e^2} k^2 \right] \tilde{A}_\mu(k, y) = -2y_c g^2 \tilde{J}_\mu(k) \delta(y), \quad (2)$$

$$\left[k^2 + \partial_y^2 \right] \tilde{A}_5(k, y) - i \partial_y \left[\delta(y) \frac{2y_c g^2}{e^2} k^\mu \tilde{A}_\mu(k, y) \right] = 0, \quad (3)$$

where we take the gauge condition, $\partial^M A_M / 2y_c g^2 + \delta(y) \partial^\mu A_\mu / e^2 = 0$. The fields with ‘tilde’s indicate the fields in (k, y) space, obtained by the Fourier transformation. In Eq. (2), the first and the second terms of the left hand side come from the bulk kinetic term in the action (1), and the third term is from the brane kinetic term localized at $y = 0$. In Eq. (2), because the right hand side is proportional only to $\delta(y)$, it is necessary to kill the first two terms in the left hand side in Eq. (2). Hence, the solution should be a combination of the trigonometric functions, $\sin ky$ and $\cos ky$. The single-valued condition of the solution on S^1 requires different combinations of

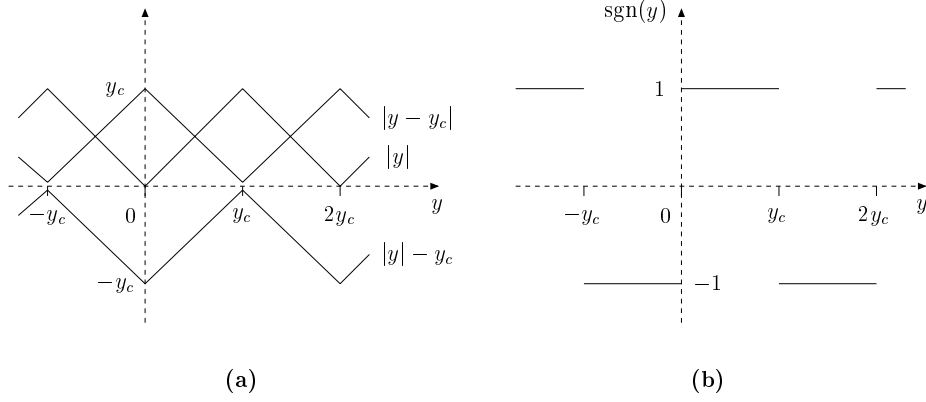


Figure 1: The definitions of the functions, **(a)** $|y - y_c|$, $|y|$, and $|y| - y_c$, and **(b)** $\text{sgn}(y)$ in this paper. They all are periodic in the compact extra dimension.

$\sin ky$ and $\cos ky$ at the two intervals $(-y_c, 0)$ and $(0, y_c)$, which potentially gives rise to singular terms proportional to $\delta(y)$ and $\delta(y - y_c)$. A unique combination that does not generate any singular term at $y = y_c$ is $\cos k|y - y_c|$, and the solution is given by

$$\tilde{A}_\mu^{b1}(k, y) = -f(k) \cos k|y - y_c| \cdot 2y_c g^2 \tilde{J}_\mu(k) , \quad (4)$$

where our definition of “ $|y - y_c|$ ” is shown in Fig. 1-(a), and $f(k)$ is determined to satisfy the equation of motion,

$$f(k) = \frac{1}{2k \sin ky_c + \frac{2g^2}{e^2} k^2 y_c \cos ky_c} . \quad (5)$$

With the solution Eq. (4), it can be also proved that $\tilde{A}_5(k, y)$ in Eq. (3) should be an odd function proportional to $\text{sgn}(y)k^\mu \tilde{A}_\mu$, where our definition of “ $\text{sgn}(y)$ ” is shown in Fig. 1-(b).

The expression of $\cos|y - y_c|$ in Eq. (4) contains Kaluza-Klein (KK) modes’ contributions. To compare the solution with the ordinary solution in the absence of the brane kinetic term, it is necessary to expand $\cos|y - y_c|$ in the fifth momentum space with the bases of $e^{ik_5 y} (= e^{in\pi y/y_c})$,

$$\cos k|y - y_c| = \frac{1}{2y_c} \sum_{n=-\infty}^{\infty} \tilde{g}_n e^{in\pi y/y_c} , \quad (6)$$

where \tilde{g}_n is given by

$$\tilde{g}_n = \int_{-y_c}^{y_c} dy e^{-in\pi y/y_c} \cos k|y - y_c| = \frac{2k \sin ky_c}{k^2 - (n\pi/y_c)^2} . \quad (7)$$

Hence, the solution in (x, y) space is

$$A_\mu^{b1}(x, y) = - \int \frac{d^4 k}{(2\pi)^4} \sum_{k_5} \frac{e^{ikx} e^{ik_5 y}}{k^2 - k_5^2} \frac{e^2 g^2 \tilde{J}_\mu(k)}{e^2 + g^2 k y_c \cot k y_c} . \quad (8)$$

We note that when both the bulk and the brane kinetic terms are present and a source is localized on the brane, the bulk gauge coupling g^2 is effectively replaced by $e^2 g^2 / (e^2 + g^2 k y_c \cot k y_c)$. In the limit $e^2 \rightarrow \infty$, which corresponds to the case in which the brane kinetic term is absent in the action (1), the solution reduces to the ordinary solution. In the low energy limit $k y_c \rightarrow 0$ (or $k y_c \cot k y_c \rightarrow 1$), the effective coupling is given by $e^2 g^2 / (e^2 + g^2)$. It is also consistently checked in the action after integrating out its y dependence. In the Euclidian space, $k y_c \cot k y_c$ becomes $k_E \coth k_E y_c$. Thus, the gauge interaction at high energy is more suppressed by the factor $e^2 / (g^2 k_E y_c)$ compared to that expected in the ordinary theory without the brane kinetic term [7].

Similarly, we can directly derive the solution for the case when a brane kinetic term and a source term are located at $y = y_c$,

$$\tilde{A}_\mu^{b2}(k, y) = - \frac{\cos k|y| \cdot 2y_c g^2 \tilde{J}_\mu(k)}{2k \sin ky_c + \frac{2g^2}{e_c^2} k^2 y_c \cos ky_c} , \quad (9)$$

where e_c is the gauge coupling of a gauge kinetic term localized at $y = y_c$. Our definition of “ $|y|$ ” is also shown in Fig.1-(a). The solution (9) does not generate a singular term proportional to $\delta(y)$. The function $\cos k|y|$ expands in the fifth momentum space in the following way:

$$\cos k|y| = \frac{1}{2y_c} \sum_{n=-\infty}^{\infty} e^{in\pi(y-y_c)/y_c} \frac{2k \sin ky_c}{k^2 - (n\pi/y_c)^2} . \quad (10)$$

We note again that for $e_c^2 \gg 1$ and $1/y_c \gg k$, the solution reduces to the ordinary one with the coupling g^2 .

Let us consider the case containing two localized sources both at $y = 0$ and at $y = y_c$, and a brane gauge kinetic term at $y = 0$,

$$\left[k^2 + \partial_y^2 + \delta(y) \frac{2y_c g^2}{e^2} k^2 \right] \tilde{A}_\mu(k, y) = -2y_c g^2 \left(\tilde{J}_\mu^{b1}(k) \delta(y) + \tilde{J}_\mu^{b2}(k) \delta(y - y_c) \right) \quad (11)$$

As a trial solution, we take $\tilde{A}_\mu(k, y) = \tilde{A}_\mu^{b1}(k, y) + \tilde{A}_\mu^{b2}(k, y)$ with $e_c \rightarrow \infty$, where $\tilde{A}_\mu^{b1}(k, y)$ and $\tilde{A}_\mu^{b2}(k, y)$ are given from Eq. (4) and (9), respectively. Then the left hand side of Eq. (11) gives

$$-2y_c g^2 \left(\tilde{J}_\mu^{b1}(k) \delta(y) + \tilde{J}_\mu^{b2}(k) \delta(y - y_c) \right) - 2y_c g^2 \delta(y) \frac{g^2}{e^2} \tilde{I}_\mu^{b2}(k) , \quad (12)$$

where $\tilde{I}_\mu^{b2}(k)$ is defined as

$$\tilde{I}_\mu^{b2}(k) \equiv \sum_{n=-\infty}^{\infty} \frac{(-1)^n k^2}{k^2 - (n\pi/y_c)^2} \tilde{J}_\mu^{b2}(k) . \quad (13)$$

The effective 4D theory at low energy could be obtained by decoupling or truncating all of heavy KK modes. Then, $\tilde{I}_\mu^{b2}(k)$ is given just by $\tilde{J}_\mu^{b2}(k)$. To cancel the unwanted third term, the source term at $y = 0$ should be shifted to $\tilde{J}_\mu^{b1}(k) - (g^2/e^2) \tilde{I}_\mu^{b2}$. Thus, the solution is

$$A_\mu(x, y) = A_\mu^{b1}(x, y) + A_\mu^{b2}(x, y) , \quad (14)$$

$$A_\mu^{b1}(x, y) = - \int \frac{d^4 k}{(2\pi)^4} \sum_n \frac{e^{ikx} e^{in\pi y/y_c}}{k^2 - (n\pi/y_c)^2} \frac{(e^2 g^2 \tilde{J}_\mu^b(k) - g^4 \tilde{I}_\mu^{b2}(k))}{e^2 + g^2 k y_c \cot k y_c} , \quad (15)$$

$$A_\mu^{b2}(x, y) = - \int \frac{d^4 k}{(2\pi)^4} \sum_n \frac{e^{ikx} e^{in\pi(y-y_c)/y_c}}{k^2 - (n\pi/y_c)^2} g^2 \tilde{J}_\mu^{b2}(k) . \quad (16)$$

We note that the two point Green functions of the gauge interaction between two brane sources at $y = 0$ are different from those between those at $y = y_c$. However, if $k y_c \ll 1$, at low energy where massive KK particles are decoupled, their effective gauge couplings ($e^2 g^2 / (e^2 + g^2)$) become universal and the propagator reduces to ordinary 4D propagator form.

Similarly, we can also easily derive the solution in the presence of a bulk source as well as a source localized at $y = 0$,

$$A_\mu(x, y) = A_\mu^B(x, y) + A_\mu^b(x, y) , \quad (17)$$

$$A_\mu^B(x, y) = - \int \frac{d^4 k}{(2\pi)^4} \sum_{k_5} \frac{e^{ikx} e^{ik_5 y}}{k^2 - k_5^2} \cdot g^2 \hat{J}_\mu^B(k, k_5) , \quad (18)$$

$$A_\mu^b(x, y) = - \int \frac{d^4 k}{(2\pi)^4} \sum_{k_5} \frac{e^{ikx} e^{ik_5 y}}{k^2 - k_5^2} \frac{(e^2 g^2 \tilde{J}_\mu^b(k) - g^4 \tilde{I}_\mu^B(k))}{e^2 + g^2 k y_c \cot k y_c} , \quad (19)$$

where $\hat{J}_\mu^B(k, k_5)$ and $\tilde{I}_\mu^B(k)$ are defined as

$$\tilde{J}_\mu^B(k, y) \equiv \frac{1}{2y_c} \sum_{k_5} \hat{J}_\mu^B(k, k_5) e^{ik_5 y} , \quad (20)$$

$$\tilde{I}_\mu^B(k) \equiv \sum_{k_5} \frac{k^2}{k^2 - k_5^2} \hat{J}_\mu^B(k, k_5) . \quad (21)$$

Thus, the propagator mediating two bulk sources and two localized sources in 4D space-time are, respectively,

$$\sum_n \frac{-ig_{\mu\nu} g^2}{k^2 - (n\pi/y_c)^2} \left(1 - \frac{g^2}{e^2 + g^2 k y_c \cot k y_c} \sum_{n'} \frac{k^2}{k^2 - (n'\pi/y_c)^2} \right) , \quad (22)$$

$$\sum_n \frac{-ig_{\mu\nu}}{k^2 - (n\pi/y_c)^2} \frac{e^2 g^2}{e^2 + g^2 k y_c \cot k y_c} . \quad (23)$$

We note again that if all KK modes are decoupled, in the limit $k y_c \rightarrow 0$, they become the same as each other,

$$\frac{e^2 g^2}{e^2 + g^2} \frac{-ig_{\mu\nu}}{k^2} , \quad (24)$$

where $e^2 g^2 / (e^2 + g^2)$ is the effective coupling at low energy. Thus, *even if g^2 is extremely large, the effective gauge couplings of the two propagators could be small if e^2 is tiny.*

When two branes at $y = 0$ and $y = y_c$ contain both a brane kinetic term and a source term, respectively, it is difficult to obtain the exact solution. However, if $g^2 \ll e_1^2, e_2^2$ or $g^2, e_1^2 \ll e_2^2$, we can get approximated solutions around the solutions when $e_1^2, e_2^2 \rightarrow \infty$ or $e_2^2 \rightarrow \infty$ which are provided already. We can check that at low energy the approximated solutions also maintain the universality of the gauge couplings.

Now let us discuss for a while the property of the function, $g^2 / (e^2 + g^2 k y_c \cot k y_c)$ appearing in Eq. (22) and (23). The function $g^2 / (e^2 + g^2 k y_c \cot k y_c)$ contains infinite number of poles, and it could be expanded with functions singular at the poles. It means that infinite number of additional massive particles are concerned in the propagator. To see it clearly, let us consider the simpler case of $e^2 = 0$. It corresponds

to the case that the effect of the brane kinetic term is extremely large. $1/ky_c \cot ky_c$ is

$$\frac{1}{ky_c \cot ky_c} = \frac{2/y_c^2}{(\pi/2y_c)^2 - k^2} + \frac{2/y_c^2}{(3\pi/2y_c)^2 - k^2} + \frac{2/y_c^2}{(5\pi/2y_c)^2 - k^2} + \cdots, \quad (25)$$

where $\pi/2y_c, 3\pi/2y_c, 5\pi/2y_c$, and so forth are singular points of $1/ky_c \cot ky_c$. Thus, at low energy $k \ll 1/y_c$, it gives $(2/y_c^2)(2y_c/\pi)^2(1+1/3^2+1/5^2+\cdots) = (8/\pi^2)(3\pi^2 B_1/4) = 1$, where $B_1 = 1/6$ is a Bernoulli number.

When we turn on e^2 , the expansion in Eq. (25) should be changed. $g^2/(e^2 + g^2 ky_c \cot ky_c)$ expands with the functions,

$$\frac{1}{ky_c \cot ky_c} \left[\left(\frac{e^2}{g^2} \right) \frac{1}{ky_c \cot ky_c} \right]^n, \quad (26)$$

in which the expansion of $1/ky_c \cot ky_c$ was given in Eq. (25). Thus, the second term in the propagator Eq. (22) expands

$$\begin{aligned} & \sum_{k_5, k'_5} \left[\frac{-ig_{\mu\nu} g^2}{k^2 - k_5^2} \frac{k^2}{k^2 - k'^2_5} \right] \left(\frac{g^2}{e^2 + g^2 ky_c \cot ky_c} \right) \\ &= \sum_{k_5, k'_5} \left[\frac{-ig_{\mu\nu} g^2}{k^2 - k_5^2} \frac{k_5^2}{k^2 - k'^2_5} - \frac{-ig_{\mu\nu} g^2}{k^2 - k'^2_5} \frac{k'^2_5}{k^2 - k_5^2} \right] \\ & \quad \times \left(\frac{1}{ky_c \cot ky_c} \left(1 - \frac{e^2}{g^2 ky_c \cot ky_c} + \cdots \right) \right). \end{aligned} \quad (27)$$

Since $(1/ky_c \cot ky_c)^n$ can be always decompsed to a summation of the functions proportional to $1/[(2m+1)\pi/2y_c]^2 - k^2$ ($m = 0, 1, 2, \cdots$), we can conclude that even when $e^2 \neq 0$ the propagator in Eq. (22) can be expressed always as the summation of the ordinary prapagators in which more heavy modes with masses $M_m = (2m+1)\pi/2y_c$ ($m = 0, 1, 2, \cdots$) are involved. Thus, at low energy $\mu < \pi/y_c$, only the mode with $k_5 = k'_5 = 0$ makes a contribution to the propagator, and the part in the parenthesis in Eq. (27) gives the modified gauge coupling $e^2/(e^2 + g^2)$ because $M_m > \pi/y_c$ always.

As seen in the case $e^2 = 0$, the case of $e^2 \neq 0$ would also introduce more massive modes with masses k'_5 and M_m . However, the above results when $e^2 = 0$ would be still approximately valid at very high energy scales beyond π/y_c even when $e^2 \neq 0$.

3 Gaugino solutions

For completeness of our discussion, let us consider the fermionic cases also. The gaugino equation of motion in the presence of the brane kinetic term as well as bulk kinetic term is

$$\left[\gamma_5 \not{k} - i\partial_y + \delta(y) \frac{2y_c g^2}{e^2} \gamma_5 \not{k} \right] \tilde{\Psi}(k, y) = 2y_c g^2 \gamma_5 \tilde{\eta}(k) \delta(y) , \quad (28)$$

where $\gamma_5 \equiv \text{diag}(iI_{2 \times 2}, -iI_{2 \times 2})$. For convenience, we multiplied γ_5 to the both side of the equation (28). The solution $\tilde{\Psi}(k, y)$ turns out to have the follownig form,

$$\tilde{\Psi}(k, y) = \tilde{\Psi}^1(k, y) - \tilde{\Psi}^2(k, y) , \quad (29)$$

$$\tilde{\Psi}^1(k, y) = f(\gamma_5 \not{k}, k) \exp\left(-i\gamma_5 \not{k}(|y| - y_c)\right) \left[\text{sgn}(y) + 1\right] \cdot 2y_c g^2 \gamma_5 \tilde{\eta}(k) , \quad (30)$$

$$\tilde{\Psi}^2(k, y) = f(\gamma_5 \not{k}, k) \exp\left(-i\gamma_5 \not{k}|y - y_c|\right) \left[\text{sgn}(y) - 1\right] \cdot 2y_c g^2 \gamma_5 \tilde{\eta}(k) , \quad (31)$$

where our definitions of “ $|y| - y_c$ ” is shown in Fig. 1-(a). This combination also does not generate any singular term at $y = y_c$. With $(\gamma_5 \not{k})^2 = k^2$, one can find the expression of $f(\gamma_5 \not{k}, k)$ satisfying the equation of motion,

$$f(\gamma_5 \not{k}, k) = \frac{\gamma_5 \not{k}}{4k} \frac{1}{\text{sink}y_c + \frac{g^2}{e^2} k y_c \text{cosk}y_c} . \quad (32)$$

Let us expand the y dependent parts of the solution, Eq. (30) and (31) in the fifth momentum space,

$$\exp\left(-i\gamma_5 \not{k}(|y| - y_c)\right) \left[\text{sgn}(y) + 1\right] = \frac{1}{2y_c} \sum_n \tilde{g}_n^1 e^{in\pi y/y_c} , \quad (33)$$

$$\exp\left(-i\gamma_5 \not{k}|y - y_c|\right) \left[\text{sgn}(y) - 1\right] = \frac{1}{2y_c} \sum_n \tilde{g}_n^2 e^{in\pi y/y_c} , \quad (34)$$

$$\tilde{g}_n^1 - \tilde{g}_n^2 = 4 \text{sink}y_c \frac{\gamma_5 \not{k}}{k} \frac{\gamma_5 \not{k} - n\pi/y_c}{k^2 - (n\pi/y_c)^2} . \quad (35)$$

Hence, the gaugino solution in (x, y) space is

$$\Psi(x, y) = \int \frac{d^4 k}{(2\pi)^4} \sum_{k_5} e^{ikx} e^{ik_5 y} \frac{\not{k} - \not{k}_5}{k^2 - k_5^2} \frac{e^2 g^2 \tilde{\eta}(k)}{e^2 + g^2 k y_c \cot k y_c} . \quad (36)$$

We note that the exactly same factor as that in the gauge field solution Eq. (8) comes out again in the gaugino solution.

Of course, we can study every configuration considered already in the gauge boson cases for the brane fermionic kinetic terms and sources again. But here let us discuss only the case that a brane gaugino kinetic term, and a bulk and a brane sources are present. Then the equation of motion for the gaugino is

$$\left[\gamma_5 \not{k} - i\partial_y + \delta(y) \frac{2y_c g^2}{e^2} \gamma_5 \not{k} \right] \tilde{\Psi}(k, y) = 2y_c g^2 \gamma_5 \left(\tilde{\eta}^B(k, y) + \tilde{\eta}^b(k) \delta(y) \right). \quad (37)$$

The solution has the similar form as the bosonic one Eq. (17)–(21),

$$\Psi(x, y) = \Psi^B(x, y) + \Psi^b(x, y), \quad (38)$$

$$\Psi^B(x, y) = \int \frac{d^4 k}{(2\pi)^4} \sum_{k_5} e^{ikx} e^{ik_5 y} \frac{\not{k} - \not{k}_5}{k^2 - k_5^2} \cdot g^2 \hat{\eta}^B(k, k_5), \quad (39)$$

$$\Psi^b(x, y) = \int \frac{d^4 k}{(2\pi)^4} \sum_{k_5} e^{ikx} e^{ik_5 y} \frac{\not{k} - \not{k}_5}{k^2 - k_5^2} \frac{(e^2 g^2 \tilde{\eta}^b(k) - g^4 \tilde{\zeta}^B(k))}{e^2 + g^2 k y_c \cot k y_c}, \quad (40)$$

where $\hat{\eta}^B(k, k_5)$ and $\tilde{\zeta}^B(k)$ are defined as

$$\tilde{\eta}^B(k, y) \equiv \frac{1}{2y_c} \sum_{k_5} \hat{\eta}^B(k, k_5) e^{ik_5 y}, \quad (41)$$

$$\tilde{\zeta}^B(k) \equiv \sum_{k_5} \frac{k^2 - k_5^2}{k^2 - k_5^2} \hat{\eta}^B(k, k_5). \quad (42)$$

4 The model

Now let us consider S^1/Z_2 orbifold compactification of the 5D, $SU(2)$ gauge theory. We assign Z_2 odd parity to charged generators T^1, T^2 , and even parity to diagonal one T^3 , which is consistent with the $SU(2)$ Lie algebra. Then $SU(2)$ gauge symmetry breaks down to $U(1)$, and only $U(1)$ is respected on the brane. In this setup, the low energy theory is just $U(1)$ gauge theory. On the other hand, above the compactification scale $\mu > 1/y_c$, $SU(2)$ symmetry is restored approximately.

In addition to this, we can introduce a gauge kinetic term on the S^1/Z_2 fixed point (brane) at $y = 0$ having the form,

$$-\delta(y) \frac{1}{4e^2} F_{\mu\nu}^3 F^{3\mu\nu}, \quad (43)$$

where $F_{\mu\nu}^3 \equiv \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3$, and ‘3’ is the group index of $SU(2)$. The additional $U(1)$ gauge kinetic term on the brane is not harmful in the model. Then, the solution of A_μ^3 is given by Eq. (17)–(21), while the solutions for A_μ^1 and A_μ^2 are the ordinary ones in the absence of the brane kinetic term, Eq. (18), since their wave functions vanish at the brane. Additionally, we introduce some matter fields only on the brane.

In this setup, we consider radiative corrections to the gauge boson masses and the gauge couplings. When we discuss radiative corrections in non-Abelian gauge theory, we should consider also the diagrams in which the ghost fields are involved in the loops. To get the ghost Lagrangian, it is necessary to examine the gauge transformation. The bulk and brane gauge kinetic terms are invariant under the gauge transformation,

$$\delta A_M^a = \frac{1}{2y_c g^2} \left[\partial_M \alpha^a - \epsilon^{abc} A_M^b \alpha^c \right] + \frac{1}{e^2} \left[\partial_M \left(\alpha^a \delta_\mu^M \delta(y) \right) - \epsilon^{abc} A_M^b \left(\alpha^c \delta_\mu^M \delta(y) \right) \right], \quad (44)$$

where α^a is the gauge parameter. Since odd (even) parity is assigned to α^1, α^2 (α^3), α^1, α^2 vanish at $y = 0$. Thus, non-vanishing term in the part proportional to $1/e^2$ in Eq. (44) is $\delta(y) \delta_\mu^M \partial_M \alpha^3 / e^2$. Then, by the standard procedure, the ghost Lagrangian is obtained,

$$\mathcal{L}_{\text{ghost}} = -\frac{1}{2y_c g^2} \bar{c}^a \partial_M^2 c^a - \epsilon^{abc} \bar{c}^a \partial^M A_M^b c^c - \delta(y) \frac{1}{e^2} \bar{c}^3 \partial_\mu^2 c^3. \quad (45)$$

Hence, the propagators of the ghost fields c^1, c^2 , and c^3 are expected to have a form similar to those of A_μ^1, A_μ^2 , and A_μ^3 , respectively.

Let us consider one loop mass corrections to massless and massive gauge bosons. The mass corrections of KK modes $A_{n,\mu}^3$ ($n \neq 0$) result from the loops by massive $A_{n,\mu}^1, A_{n,\mu}^2, c_n^1$, and c_n^2 as seen in Fig. 2-(a), as well as matter fields living on the brane. Since odd parity is assigned to such gauge bosons and ghost fields, their propagators are the same exactly as those in the absence of the additional brane $U(1)$ gauge kinetic term. As shown in Ref. [8], all of quadratic divergences for the masses of $A_{n,\mu}^3$ cancel, but logarithmically divergent mass corrections to *massive* KK modes do not vanish,

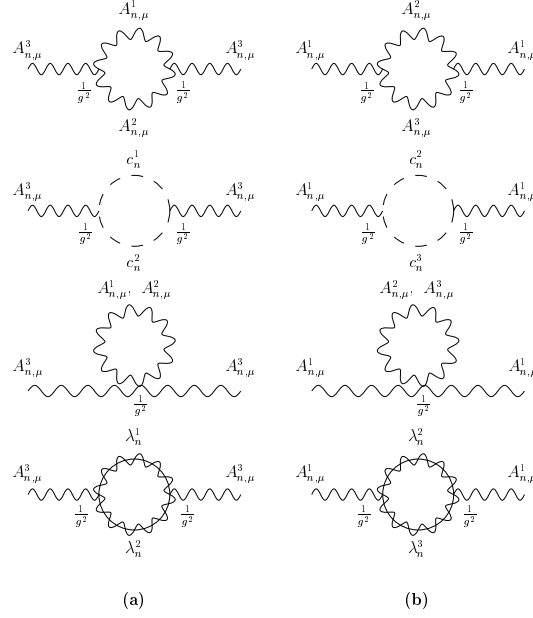


Figure 2: One loop corrections to the wave functions (or gauge couplings) and masses of (a) $A^3_{n,\mu}$, and (b) $A^1_{n,\mu}$. The third diagrams in (a) and (b) contribute only to the mass corrections of $A^3_{n,\mu}$ and $A^1_{n,\mu}$, respectively.

because the massive KK modes are not protected by gauge symmetry against such divergences. By the gauge symmetry, the massless mode $A_{0,\mu}$ remains massless [8].

The masses of $A^{1(2)}_{n,\mu}$ are corrected by $A^2_{n,\mu}$, $A^3_{n,\mu}$, c^2_n , and c^3_n as in Fig. 2-(b). Since all of the self-interacting couplings of $SU(2)$ gauge fields are contained in the bulk kinetic term $F^a_{MN}F^{aMN}$, the propagator of $A^3_{n,\mu}$ should be given by Eq. (22) rather than Eq. (23). Due to the second term in Eq. (22), the loop corrections to masses of $A^1_{n,\mu}$ and $A^2_{n,\mu}$ are different from those of $A^3_{n,\mu}$. However, the second term in the propagator, Eq. (27), does not give rise to quadratic divergences in the one loop mass corrections. Therefore, only if the cutoff scale is not very high above the compactification scale, could the KK modes masses be perturbatively stable.

Next, let us examine RG running behavior of the gauge couplings. The running behavior of g^2_{eff} is different from g^2 , because basically they are independent gauge couplings. The radiative corrections to g^2_{eff} arise from the first two diagrams in Fig. 2-(a) and the diagram by brane matter. Below the compactification scale, only $A^3_{0,\mu}$

is a massless gauge boson, and it only couples to matter. However, above the compactification scale, KK modes, $A_{n,\mu}^1$ and $A_{n,\mu}^2$ ($n \neq 0$) also take part in its correction [9]. This interaction could draw the effective gauge coupling rapidly to much smaller value at higher energy. However, we could imagine that linearly asymptotically free behavior of g_{eff}^2 above the compactification scale by KK gauge bosons could be mild by introducing many fields on the brane.

On the other hand, the propagations of $A_{n,\mu}^1$ and $A_{n,\mu}^2$ are associated only with g^2 . Above the compactification scale, g^2 also would falls down rapidly to smaller value at higher energy scale by the gauge bosons and the ghost fields contributions to the loops in Fig. 2-(b).

We can generalize our scenario to a supersymmetric system by introducing superpartners in the bulk and on the brane. We introduce another Z'_2 to break $N = 2$ SUSY into $N = 1$. As in the case of the gauge bosons, the propagation of the gauginos Ψ_n^1 and Ψ_n^2 still respect g^2 , while that of Ψ_0^3 does g_{eff}^2 at low energy. If supersymmetric chiral matter fields are assumed to be only on the brane, the blow-up of the g_{eff}^2 at low energy could be mild, but they would not affect the g^2 . Actually, the smallness of g_{eff}^2 is protected by e^2 . Thus, possibly, g_{eff}^2 remains small at low energy, while g^2 blows up above the compactification scale, $1/y_c < \mu < \Lambda_{\text{cutoff}}$. Then, Ψ^1 and Ψ^2 could condense, but Ψ^3 still composes an $N = 1$ supermultiplet together with A_μ^3 . Hence, $U(1)$ gauge symmetry and $N = 1$ supersymmetry are expected to survive at low energy. However, the condensations of Ψ_n^1 and Ψ_n^2 lead to breaking of remainig $N = 1$ SUSY softly by gravity mediation. For example, in the off-shell formalism of 5D supergravity [10],

$$\mathcal{L}_{SUGRA} = \frac{1}{2y_c} \left[\cdots - 12\vec{t}^2 + \cdots - \kappa \bar{\Psi}^a \vec{\tau} \Psi^a \vec{t} + \cdots \right], \quad (46)$$

in which κ is defined as the Planck scale mass parameters, $\kappa \equiv 1/M_P$. $\vec{\tau}$ indicates $SU(2)_R$ generators. \vec{t} is a triplet auxiliary field in the gravity multiplet. After eliminating the auxiliary fields by using its equation of motion, it is shown explicitly that the mass term of Ψ^3 is generated only if $\langle \Psi^1 \vec{\tau} \Psi^1 \rangle = \langle \Psi^2 \vec{\tau} \Psi^2 \rangle \neq 0$. SUSY breaking of

the gauge sector leads to SUSY breaking of the chiral multiplet on the brane.

Hence, in this scenario, we don't have to introduce an additional hidden sector. SUSY of the visible sector could be broken softly without an additional hidden sector, because the fields with odd parity play the role of the confining hidden sector in the gravity mediation SUSY breaking scenario. This scenario works because basically the interactions by the odd parity gauge fields, and by the even parity gauge fields are different, due to the additional brane kinetic term.

5 Conclusion

In conclusion, we obtain various particular solutions of the gauge field and gaugino in the situation where there are additional brane kinetic terms as well as the bulk gauge kinetic terms. While the gauge interaction at low energy is universal, above the compactification scale the gauge interactions on the brane and in the bulk are different. Since $N = 2$ supersymmetric $SU(2)$ gauge theory reduces to an $N = 1$ supersymmetric $U(1)$ theory at low energy by $S^1/(Z_2 \times Z'_2)$ orbifolding, we could introduce additional brane kinetic terms respecting $N = 1$ SUSY and $U(1)$ gauge symmetry at a Z_2 fixed point. In such a model, even if the bulk gauge coupling g^2 , which is asymptotically free, blew up above compactification scale, the low energy effective $U(1)$ gauge coupling $e^2 g^2 / (e^2 + g^2)$ could remain small, because they are basically independent parameters. Since the heavy gauginos with odd parity under Z_2 interact with g^2 , the blowing up of g^2 could make them condense. By gravity mediation, the effect triggers the soft mass terms of the gaugino with even parity.

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